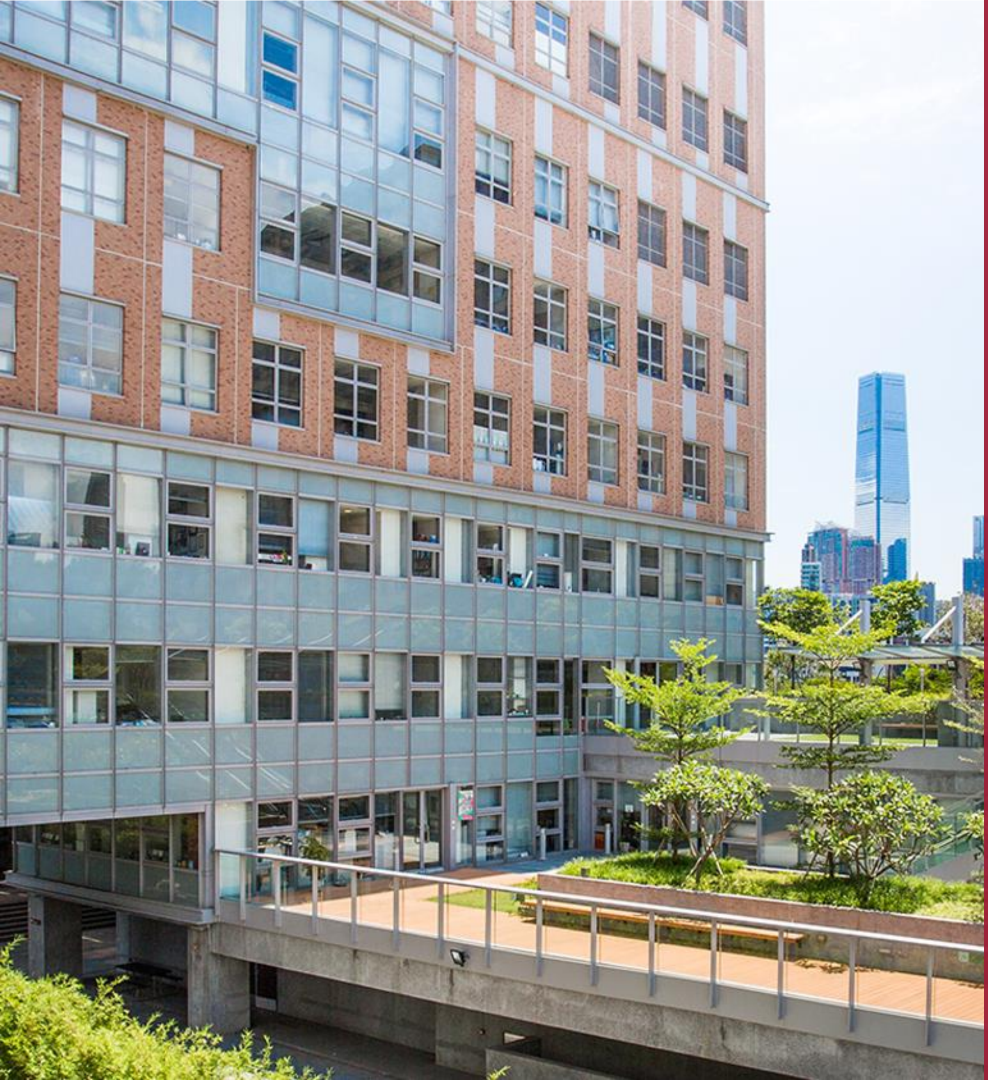

THE HONG KONG POLYTECHNIC UNIVERSITY
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Optimal Driving for Vehicle Fuel Economy under Traffic Speed Uncertainty

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Joint work with
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[Wu, F., Bektaş, T., Dong, M., Ye, H., Zhang, D., 2021. Optimal driving for vehicle fuel economy under traffic speed uncertainty. Transportation Research Part B 154, 175-206.]

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Scenario

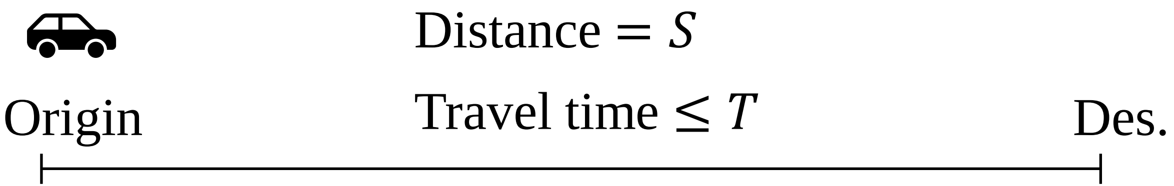
- Driving a car from origin to destination
- Route is fixed
- Need to arrive at the destination before certain time (e.g., deliver or pick up a parcle at scheduled time, attend an appointment, etc.)

Introduction: Deterministic Eco-driving

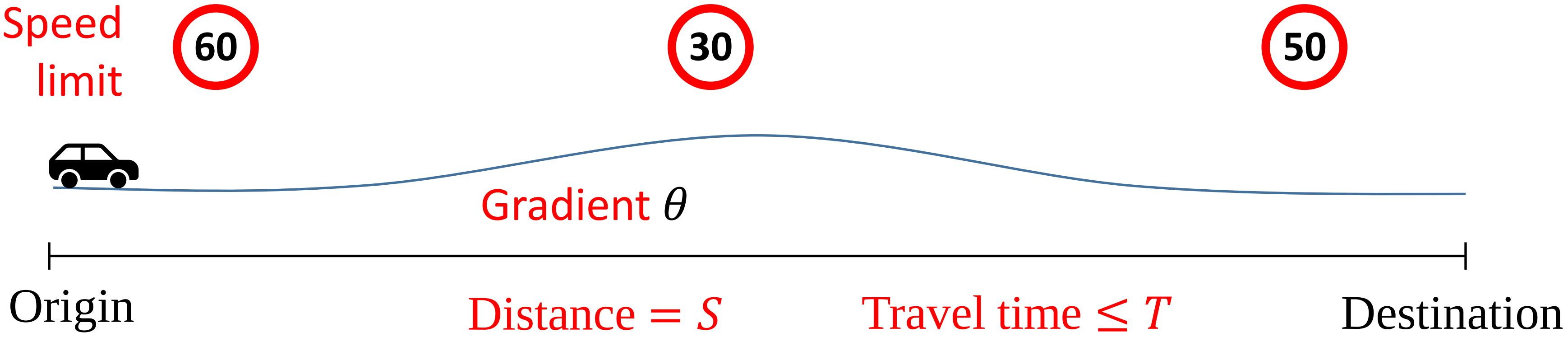
Problem/Objective:

How to drive the car, so as to

- Arrive at the destination on time, while
- Consuming minimal amount of fuel



Factors considered in eco-driving



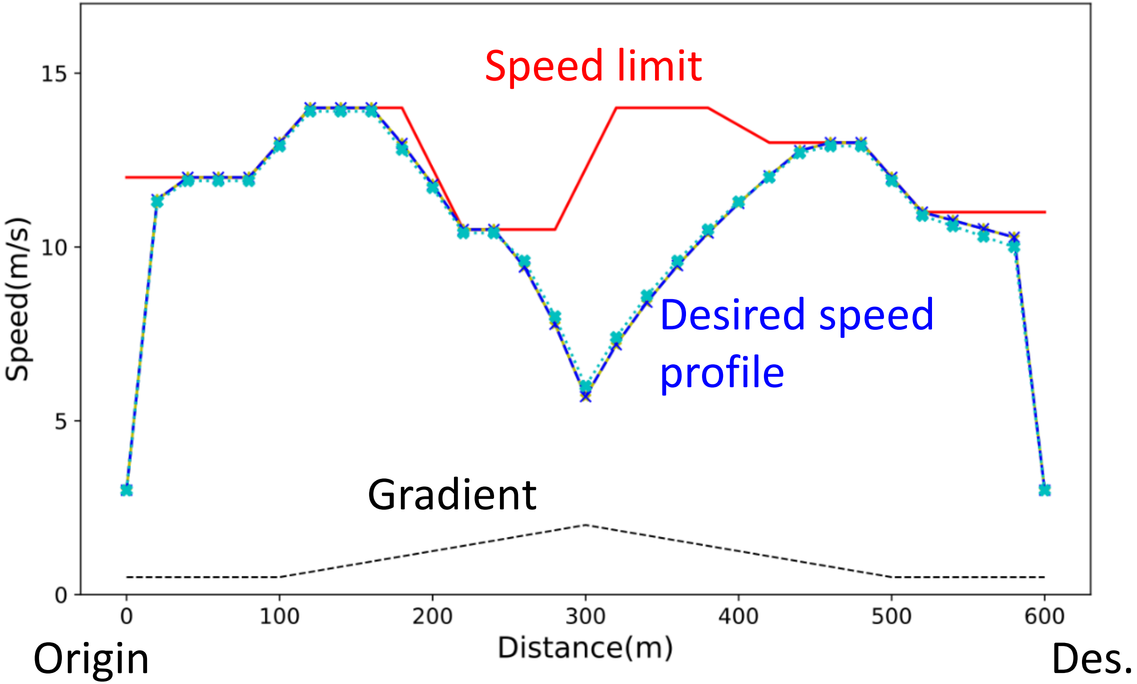
Fuel consumption rate $FR(v, a)$ (gram per second)

- $FR(\boldsymbol{v}, \boldsymbol{a}) = C_1 + C_2 \boldsymbol{v} \max\{\boldsymbol{a} + C_3 \boldsymbol{v}^2 + C_4 \cos\theta + C_5 \sin\theta, 0\}$
- v: speed; a: acceleration; $C_1 \sim C_5$: parameters

Introduction: Deterministic Eco-driving

Solving the deterministic eco-driving problem, we can get a speed profile that

- Guarantees the vehicle reaching destination on time,
- Minimizes the fuel consumption



Introduction: Eco-driving under Uncertain Traffic Speed

- Movement of our vehicle can be blocked by other vehicles in front
- Our vehicle cannot drive faster than traffic speed (usually uncertain)
- How to solve eco-driving under uncertain traffic speed?

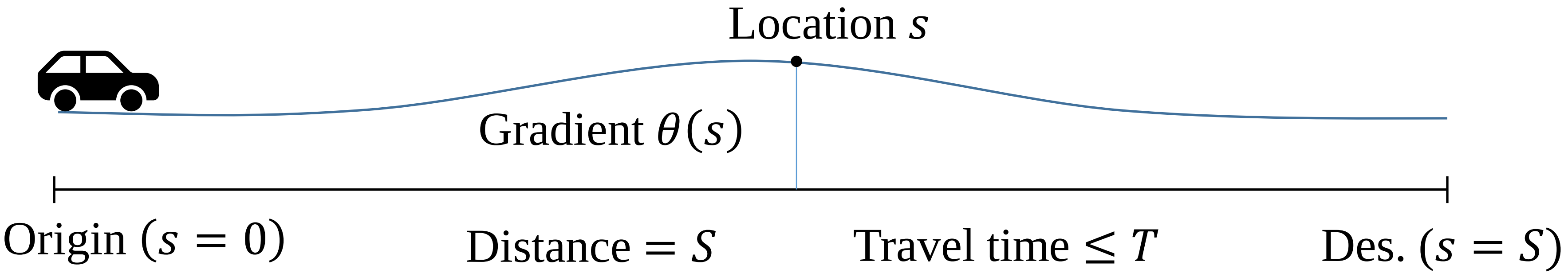
What We Did in This Research

1. Propose a new model to solve the deterministic eco-driving problem much more efficiently (by converting a non-convex program to a mixed-integer linear program)
2. Formulate the eco-driving problem under uncertain traffic speed as stochastic optimization problems
3. Solve the stochastic optimization problems in Step 2 using sample average approximation (SAA)

Deterministic Eco-driving Model

- At location s, denote clock time as $t(s)$ and speed as $v(s)$

$$v(s) = \frac{\mathrm{d}s}{dt} \Rightarrow \mathrm{d}t = \frac{1}{v(s)}\mathrm{d}s \Rightarrow \begin{cases} \text{Acceleration } a(s) = \dfrac{\mathrm{d}v(s)}{\mathrm{d}t(s)} = \dfrac{\mathrm{d}v(s)}{\mathrm{d}s}v(s) = \dfrac{\mathrm{d}\left(v(s)\right)^2}{2\mathrm{d}s} \\ \text{Total trip time} = \displaystyle\int_0^S \mathrm{d}t = \int_0^S \frac{1}{v(s)}\mathrm{d}s \end{cases}$$



Deterministic Eco-driving Model

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

s.t. $$\int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s\in[0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s\in[0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s\in[0,S]$$

$$v(0) = V_0, v(S) = V_S$$

Optimal control formulation

Deterministic Eco-driving Model

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

Total fuel consumption of the trip

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

S: length of the journey

s: location

$v(s)$: speed at location s

$a(s)$: acceleration at location s

$FR(v,a)$: fuel assumption rate

$$\mathrm{d}t = \frac{1}{v(s)}\mathrm{d}s$$

Deterministic Eco-driving Model

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

s.t. $\int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$ Trip time constraint

$$a(s)=\frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s\in[0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s\in[0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s\in[0,S]$$

$$v(0)=V_0, v(S)=V_S$$

S: length of the journey

s: location

$v(s)$: speed at location s

$$\mathrm{d}t=\frac{1}{v(s)}\mathrm{d}s$$

Deterministic Eco-driving Model

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

Speed limit constraint

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

$v(s)$: speed at location s

$\bar{V}(s)$: speed limit at location s

ε: small positive number

Deterministic Eco-driving Model

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

Accl. capacity

$$v(0) = V_0, v(S) = V_S$$

$a(s)$: acceleration at location s

$\underline{a}$ & $\bar{a}$: lower & upper bound of accl.

Deterministic Eco-driving Model

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

V_0 & V_S: desired speed at origin & des.

Solution Method – Discretization

$$\min_{v(s),a(s):s\in[0,S]} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\,\mathrm{d}s$$

Optimal control formulation

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\,\mathrm{d}s \le T$$

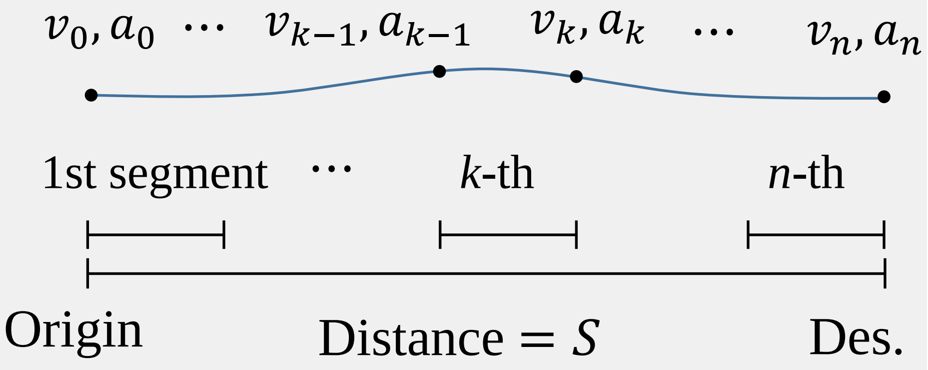
$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

Discretize S into n uniform segments.
Length of each segment $\Delta s = S/n$



Solution Method – Non-Convex Programming

$$\min_{v(s),a(s)} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \quad \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

Optimal control formulation

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

$$\min_{v_k,\, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k)\frac{\Delta s}{v_k}$$

$$\text{s.t.} \quad \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$$

Non-convex program

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}, k = 0,1,\cdots,n-1$$

$$\varepsilon \le v_k \le \bar{V}_k, k = 0,1,\cdots,n-1$$

$$\underline{a} \le a_k \le \bar{a}, k = 0,1,\cdots,n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Non-Convex Programming

$$\min_{v(s),a(s)} \int_0^S FR\big(v(s), a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0, S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0, S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0, S]$$

$$v(0) = V_0, v(S) = V_S$$

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k)\frac{\Delta s}{v_k}$$

$$\text{s.t.} \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$$

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$$\underline{a} \le a_k \le \bar{a}, k = 0,1,\cdots,n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Non-Convex Programming

$$\min_{v(s),a(s)} \int_0^S FR\big(v(s), a(s)\big) \frac{1}{v(s)} \mathrm{ds}$$

$$\text{s.t.} \quad \int_0^S \frac{1}{v(s)} \mathrm{ds} \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{ds}}, s \in [0, S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0, S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0, S]$$

$$v(0) = V_0, v(S) = V_S$$

⇒

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

$$\text{s.t.} \quad \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}, k = 0,1, \cdots, n-1$$

$$\varepsilon \le v_k \le \bar{V}_k, k = 0,1, \cdots, n-1$$

$$\underline{a} \le a_k \le \bar{a}, k = 0,1, \cdots, n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Non-Convex Programming

$$\min_{v(s),a(s)} \int_0^S FR(v(s),a(s))\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\left(v(s)\right)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

$$\min_{v_k,a_k} \sum_{k=0}^{n-1} FR(v_k,a_k)\frac{\Delta s}{v_k}$$

$$\text{s.t.} \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}, k = 0,1,\cdots,n-1$$

$$\varepsilon \le v_k \le \bar{V}_k, k = 0,1,\cdots,n-1$$

$$\underline{a} \le a_k \le \bar{a}, k = 0,1,\cdots,n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Non-Convex Programming

$$\min_{v(s),a(s)} \int_0^S FR\big(v(s),a(s)\big)\frac{1}{v(s)}\mathrm{d}s$$

$$\text{s.t.} \quad \int_0^S \frac{1}{v(s)}\mathrm{d}s \le T$$

$$a(s) = \frac{\mathrm{d}\big(v(s)\big)^2}{2\mathrm{d}s}, s \in [0,S]$$

$$\varepsilon \le v(s) \le \bar{V}(s), s \in [0,S]$$

$$\underline{a} \le a(s) \le \bar{a}, s \in [0,S]$$

$$v(0) = V_0, v(S) = V_S$$

$$\min_{v_k,\,a_k} \sum_{k=0}^{n-1} FR(v_k,a_k)\frac{\Delta s}{v_k}$$

$$\text{s.t.} \quad \sum_{k=0}^{n-1}\frac{\Delta s}{v_k} \le T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}, k = 0,1,\cdots,n-1$$

$$\varepsilon \le v_k \le \bar{V}_k, k = 0,1,\cdots,n-1$$

$$\underline{a} \le a_k \le \bar{a}, k = 0,1,\cdots,n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Non-Convex Programming

This non-convex program can be solved by:

- Dynamic programming
- Nonlinear programing

Issues:

- Computation speed
- Global optimality

$$\min_{v_k,\, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

$$\text{s.t.} \quad \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \leq T$$

Non-convex program

$$\boldsymbol{a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}}, k = 0,1,\cdots,n-1$$

$$\varepsilon \leq v_k \leq \bar{V}_k, k = 0,1,\cdots,n-1$$

$$\underline{a} \leq a_k \leq \bar{a}, k = 0,1,\cdots,n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Mixed Integer Linear Programming (1)

$$\min_{E_k, a_k} \sum_{k=0}^{n-1} FR\left(\sqrt{2E_k}, a_k\right) \frac{\Delta s}{\sqrt{2E_k}}$$

$$\text{s.t.} \sum_{k=0}^{n-1} \frac{\Delta s}{\sqrt{2E_k}} \leq T$$

$$a_k = \frac{E_{k+1} - E_k}{\Delta s}$$

$$\varepsilon^2/2 \leq E_k \leq \bar{V}_k^2/2$$

$$\underline{a} \leq a_k \leq \bar{a}$$

$$E_0 = V_0^2/2, E_n = V_n^2/2$$

Let $E_k = \frac{1}{2} v_k^2$

$v_k = \sqrt{2E_k}$

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

$$\text{s.t.} \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \leq T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}, k = 0,1,\cdots,n-1$$

$$\varepsilon \leq v_k \leq \bar{V}_k, k = 0,1,\cdots,n-1$$

$$\underline{a} \leq a_k \leq \bar{a}, k = 0,1,\cdots,n-1$$

$$v_0 = V_0, v_n = V_S$$

Solution Method – Mixed Integer Linear Programming (2)

$$\min_{E_k, a_k} \sum_{k=0}^{n-1} \boldsymbol{FR}\left(\sqrt{2E_k}, a_k\right) \frac{\Delta s}{\sqrt{2E_k}}$$

$$\text{s.t.} \sum_{k=0}^{n-1} \frac{\Delta s}{\sqrt{2E_k}} \le T$$

$$a_k = \frac{E_{k+1} - E_k}{\Delta s}$$

$$\underline{a} \le a_k \le \bar{a}$$

$$\varepsilon^2/2 \le E_k \le \bar{V}_k^2/2$$

$$E_0 = V_0^2/2 \, , E_n = V_n^2/2$$

Substitute

$$FR(v, a) = C_1 + C_2 v \max\{a + C_3 v^2 + +C_4 \cos\theta + C_5 \sin\theta \, , 0\}$$

$$\Delta s \sum_{k=0}^{n-1} \left[\frac{C_1}{\sqrt{2E_k}} + C_2 \max\{a_k + 2C_3 E_k + C_4 \cos\theta_k + C_5 \sin\theta_k \, , 0\} \right]$$

Solution Method – Mixed Integer Linear Programming (3)

$$\min_{E_k, a_k} \Delta s \sum_{k=0}^{n-1} \left[\frac{C_1}{\sqrt{2E_k}} + C_2 \, \underline{\boldsymbol{\max\{a_k + 2C_3E_k + C_4\cos\theta_k + C_5\sin\theta_k, 0\}}} \right]$$

Replaced by new variable y_k, plus additional constraints below

$$y_k \geq a_k + 2C_3E_k + C_4\cos\theta_k + C_5\sin\theta_k$$

$$y_k \geq 0$$

s.t. $\sum_{k=0}^{n-1} \frac{\Delta s}{\sqrt{2E_k}} \leq T$

$$a_k = \frac{E_{k+1} - E_k}{\Delta s}$$

$$\underline{a} \leq a_k \leq \bar{a}$$

$$\varepsilon^2/2 \leq E_k \leq \bar{V}_k^2/2$$

$$E_0 = V_0^2/2, E_n = V_n^2/2$$

Solution Method – Mixed Integer Linear Programming (4)

$$\min_{E_k, a_k} \Delta s \sum_{k=0}^{n-1} \left[\frac{C_1}{\sqrt{2E_k}} + C_2 y_k \right]$$

s.t. $\sum_{k=0}^{n-1} \frac{\Delta s}{\sqrt{2E_k}} \leq T$

$$a_k = \frac{E_{k+1} - E_k}{\Delta s}$$

$$\underline{a} \leq a_k \leq \bar{a}$$

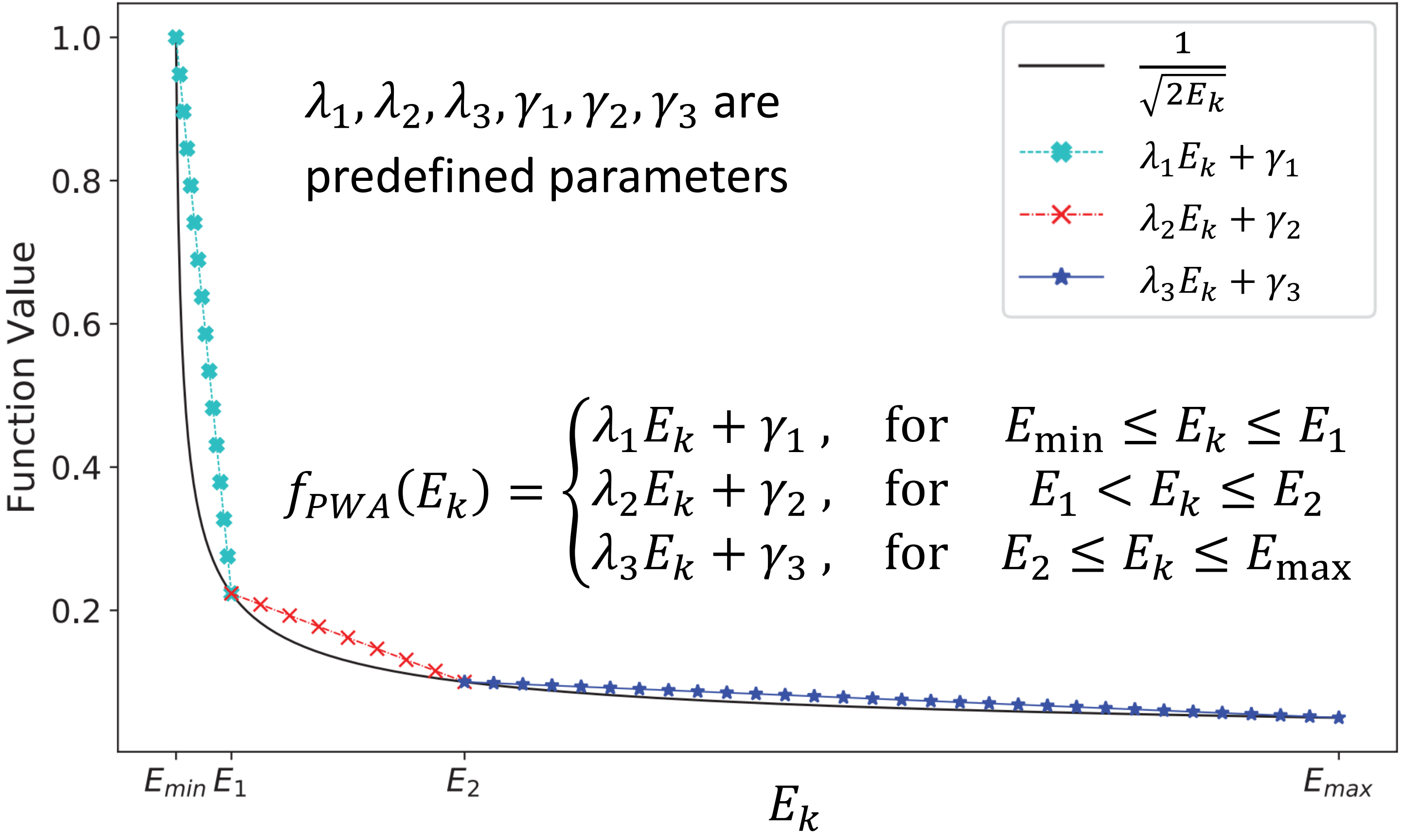
$$\varepsilon^2/2 \leq E_k \leq \bar{V}_k^2/2$$

$$E_0 = V_0^2/2 \, , E_n = V_n^2/2$$

$$y_k \geq a_k + 2C_3 E_k + C_4 \cos\theta_k + C_5 \sin\theta_k \, , y_k \geq 0$$

To linearize $\frac{1}{\sqrt{2E_k}}$, we approximate it by a **piecewise affine function $f_{PWA}(E_k)$**

Solution Method – Piecewise Affine Function



$\lambda_1, \lambda_2, \lambda_3, \gamma_1, \gamma_2, \gamma_3$ are predefined parameters

$$f_{PWA}(E_k) = \begin{cases} \lambda_1 E_k + \gamma_1\,, & \text{for} \quad E_{\min} \leq E_k \leq E_1 \\ \lambda_2 E_k + \gamma_2\,, & \text{for} \quad E_1 < E_k \leq E_2 \\ \lambda_3 E_k + \gamma_3\,, & \text{for} \quad E_2 \leq E_k \leq E_{\max} \end{cases}$$

More pieces

⇓

More accurate approximation

Solution Method – Mixed Integer Linear Programming (5)

$$\min_{E_k, a_k} \Delta s \sum_{k=0}^{n-1} [C_1 \cdot \boldsymbol{f_{PWA}(E_k)} + C_2 y_k]$$

Still nonlinear

s.t. $$\sum_{k=0}^{n-1} \Delta s \cdot \boldsymbol{f_{PWA}(E_k)} \le T$$

$$a_k = \frac{E_{k+1} - E_k}{\Delta s}$$

$$f_{PWA}(E_k) = \begin{cases} \lambda_1 E_k + \gamma_1, & \text{for} \quad E_{\min} \le E_k \le E_1 \\ \lambda_2 E_k + \gamma_2, & \text{for} \quad E_1 < E_k \le E_2 \\ \lambda_3 E_k + \gamma_3, & \text{for} \quad E_2 \le E_k \le E_{\max} \end{cases}$$

$$\underline{a} \le a_k \le \bar{a}$$

$$\varepsilon^2/2 \le E_k \le \bar{V}_k^2/2$$

$$E_0 = V_0^2/2, E_n = V_n^2/2$$

$$y_k \ge a_k + 2C_3 E_k + C_4 \cos\theta_k + C_5 \sin\theta_k, y_k \ge 0$$

Linearizing the Piecewise Affine Function

$$f_{PWA}(E_k) = -\lambda_3 z_{1,k} + (\lambda_2 - \lambda_3) z_{2,k} + (\lambda_1 - \lambda_2 + \lambda_3) z_{3,k} - \gamma_3 \delta_{1,k} + (\gamma_2 - \gamma_3)\delta_{2,k} + (\gamma_1 - \gamma_2 + \gamma_3)\,\delta_{3,k} + \lambda_3 E_k + \gamma_3$$

s.t.

$$E_k \le (E_{max} - E_i)(1 - \delta_{i,k}) + E_i\,,\ \ i \in \{1,2\}$$

$$E_k \ge E_i + \mu + (E_{min} - E_i - \mu)\delta_{i,k}\,,\ \ i \in \{1,2\}$$

$$-\delta_{i,k} + \delta_{3,k} \le 0\,,\ \ i \in \{1,2\}$$

$$\delta_{1,k} + \delta_{2,k} - \delta_{3,k} \le 1$$

$$z_{j,k} \le E_{max}\delta_{j,k}\,,\ \ j \in \{1,2,3\}$$

$$z_{j,k} \ge E_{min}\delta_{j,k}\,,\ \ j \in \{1,2,3\}$$

$$z_{j,k} \le E_k - E_{min}(1 - \delta_{j,k}),\ \ j \in \{1,2,3\}$$

$$z_{j,k} \ge E_k - E_{max}(1 - \delta_{j,k}),\ \ j \in \{1,2,3\}$$

$z_{j,k}$: new continuous variables

$\delta_{j,k}$: new binary variables

μ: sufficiently small constant

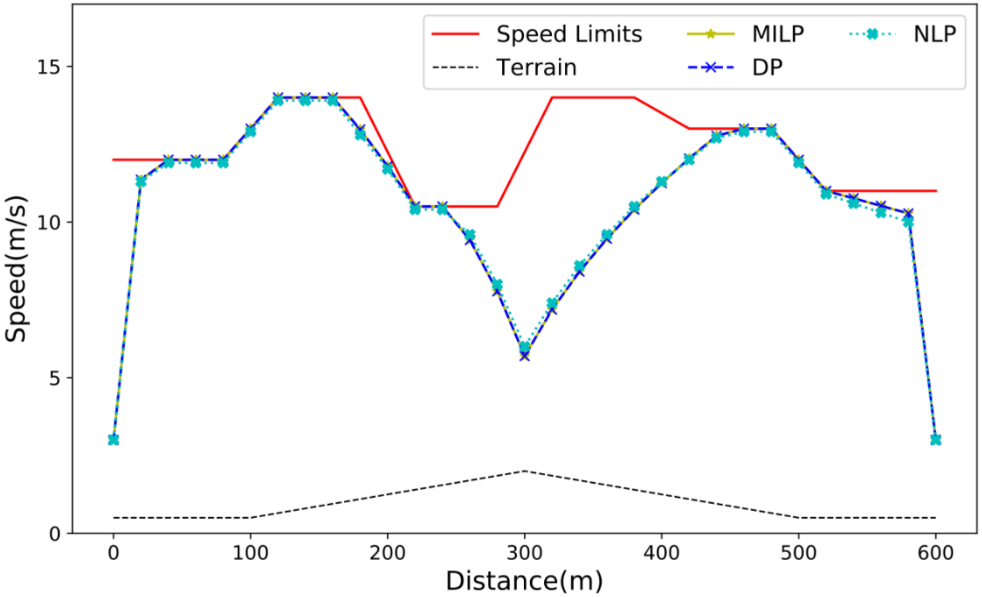
$$f_{PWA}(E_k) = \begin{cases} \lambda_1 E_k + \gamma_1\,, & \text{for} \quad E_{\min} \le E_k \le E_1 \\ \lambda_2 E_k + \gamma_2\,, & \text{for} \quad E_1 < E_k \le E_2 \\ \lambda_3 E_k + \gamma_3\,, & \text{for} \quad E_2 \le E_k \le E_{\max} \end{cases}$$

Case Study: Comparing Different Solution Methods

- Case setting
  - Distance $S = 600m$, discretized to 30 segments
  - Travel time budget $T = 61s$
- Solution methods compared
  - DP: Dynamic programming on the non-convex program
  - NLP: Nonlinear programming on the non-convex program
  - MILP (mixed integer linear program): $\frac{1}{\sqrt{2E_k}}$ linearized to 50 pieces
  - Programmed in Python
  - NLP and MILP solved by Gurobi 9.0

Case Study: Different Solution Methods ($T = 61$s)

- DP: Dynamic programming
- NLP: Nonlinear programming
- MILP: Mixed-inter linear programming



	Trip duration (s)	Fuel used (g)	Computing time (s)
NLP	61.00	113.48	1338
MILP	60.97	113.49	0.30
DP	60.85	114.10	62

Recall: Uncertain Traffic Speed

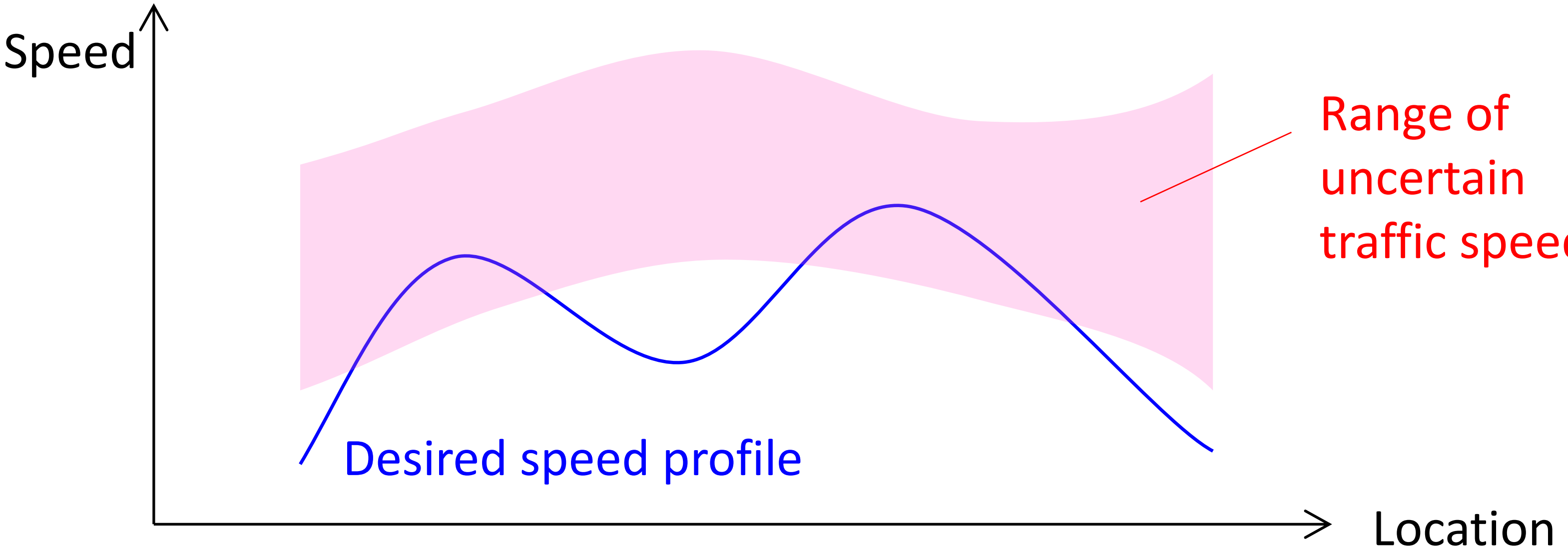
- Movement of our vehicle can be blocked by other vehicles in front
- Our vehicle cannot drive faster than traffic speed (usually uncertain)
- How to solve eco-driving under uncertain traffic speed?

Origin

Destination

Eco-driving under Uncertain Traffic Speed

- If the realized traffic speed is lower than the desired speed, driver has to follow traffic speed and cannot follow the desired speed
- This increases travel time and leads to **late arrival at the destination**



Recall: Deterministic Eco-driving Model

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

$$\text{s.t.} \quad \sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \leq T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$$\varepsilon \leq v_k \leq \overline{V}_k, k = 0,1,\cdots,n-1$$

$$\underline{a} \leq a_k \leq \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- The uncertain traffic speed serves as speed limits on the vehicle
- So we assume **the speed limits $\overline{V}_k$ to be random variables with known distribution**

Stochastic Eco-driving: Model 1

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

s.t. $\sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

random

$$\varepsilon \le \boldsymbol{v_k} \le \overline{\boldsymbol{V}}_{\boldsymbol{k}}, k = 0,1,\cdots,n-1$$

$$\underline{a} \le a_k \le \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- Chance constraint: At each location of the trip, the probably that the desired speed is achievable is $\ge 1-\alpha$

$$\begin{cases} \boldsymbol{Prob(v_k \le \overline{V}_k) \ge 1-\alpha, \forall k} \\ v_k \ge \varepsilon \end{cases}$$

- Can be converted to the following deterministic constraint using the cumulative distribution function $F_k(\cdot)$

$$\boldsymbol{v_k \le F_k^{-1}(\alpha), \forall k}$$

Stochastic Eco-driving: Model 1

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

s.t. $\sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$$\varepsilon \le \boldsymbol{v_k} \le \boldsymbol{\bar{V}_k}, k = 0,1,\cdots,n-1$$

$$\underline{a} \le a_k \le \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- **Pro**: deterministic optimization, easy to solve
- **Con**: does not reflect/consider the impact on actual travel time and fuel consumption

Replaced by → $\varepsilon \le \boldsymbol{v_k} \le \boldsymbol{F_k^{-1}(\alpha), \forall k}$

Stochastic Eco-driving: Model 2

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

s.t. $$\sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$$\varepsilon \le \boldsymbol{v_k} \le \boldsymbol{\bar{V}_k}$$

$$\underline{a} \le a_k \le \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- Define our vehicle's real speed

$$v_k^{real} = \min\{v_k, \boldsymbol{\bar{V}_k}\}$$

Desired speed (v_k)

Random traffic speed ($\bar{V}_k$)

Stochastic Eco-driving: Model 2

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

s.t. $$\sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \leq T$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$$\varepsilon \leq \boldsymbol{v_k} \sout{\leq \overline{\boldsymbol{V}}_{\boldsymbol{k}}}$$

$$\underline{a} \leq a_k \leq \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- Our vehicle's real speed $v_k^{real} = \min\{v_k, \overline{\boldsymbol{V}}_{\boldsymbol{k}}\}$

$$\min_{v_k, a_k} \mathbb{E}\left[\sum_{k=0}^{n-1} FR\left(v_k^{real}, a_k^{real}\right) \frac{\Delta s}{v_k}\right]$$

$$a_k^{real} = \frac{\left(v_{k+1}^{real}\right)^2 - \left(v_k^{real}\right)^2}{2\Delta s}$$

$$Prob\left(\sum_{k=0}^{n-1} \frac{\Delta s}{v_k^{real}} \leq T\right) \geq 1 - \alpha$$

Stochastic Eco-driving: Model 2

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

s.t. $\sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \le T$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$\varepsilon \le \boldsymbol{v_k}$ ~~$\le \bar{\boldsymbol{V}}_{\boldsymbol{k}}$~~

$$\underline{a} \le a_k \le \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- Our vehicle's real speed

$$v_k^{real} = \min\{v_k, \bar{\boldsymbol{V}}_{\boldsymbol{k}}\}$$

- Chance constraint: The probability that the actual trip time being $\le T$ is $\ge 1 - \alpha$

$$Prob\left(\sum_{k=0}^{n-1} \frac{\Delta s}{v_k^{real}} \le T\right) \ge 1 - \alpha$$

Stochastic Eco-driving: Model 2

$$\min_{v_k, a_k} \sum_{k=0}^{n-1} FR(v_k, a_k) \frac{\Delta s}{v_k}$$

s.t. $\sum_{k=0}^{n-1} \frac{\Delta s}{v_k} \leq T$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$$\varepsilon \leq \boldsymbol{v_k} \leq \boldsymbol{\overline{V}_k}$$

$$\underline{a} \leq a_k \leq \bar{a}$$

$$v_0 = V_0, v_n = V_S$$

- Our vehicle's real speed

$$v_k^{real} = \min\{v_k, \boldsymbol{\overline{V}_k}\}$$

$$\min_{v_k, a_k} \mathbb{E}\left[\sum_{k=0}^{n-1} FR\left(v_k^{real}, a_k^{real}\right) \frac{\Delta s}{v_k^{real}}\right]$$

$$a_k^{real} = \frac{\left(v_{k+1}^{real}\right)^2 - \left(v_k^{real}\right)^2}{2\Delta s}$$

- Minimize the expected actual fuel consumption evaluated using the real speed and real acceleration

Stochastic Eco-driving: Model 2 (Relaxed Form)

- Model 2 is reformulated to the stochastic optimization problem below with relaxed chance constraint, and solved using Sample Average Approximation (SAA)

$$\min_{v_k, a_k, x_k} \mathbb{E} \sum_{k=0}^{n-1} FR\left(v_k^{real}, a_k^{real}\right) \frac{\Delta s}{v_k^{real}}$$

s.t.

$$v_k^{real} = \min\{v_k, \bar{V}_k\}$$

$$a_k^{real} = \frac{\left(v_{k+1}^{real}\right)^2 - \left(v_k^{real}\right)^2}{2\Delta s}$$

$$a_k = \frac{v_{k+1}^2 - v_k^2}{2\Delta s}$$

$$\varepsilon \le v_k$$

$$\underline{a} \le a_k \le \bar{a}$$

$$\sum_{k=0}^{n-1} x_k = T$$

$$v_k x_k \ge \Delta s, \forall k$$

$$Prob(\bar{V}_k x_k \ge \Delta s) \ge 1 - \alpha, \forall k$$

$$v_0 = V_0, v_n = V_S$$

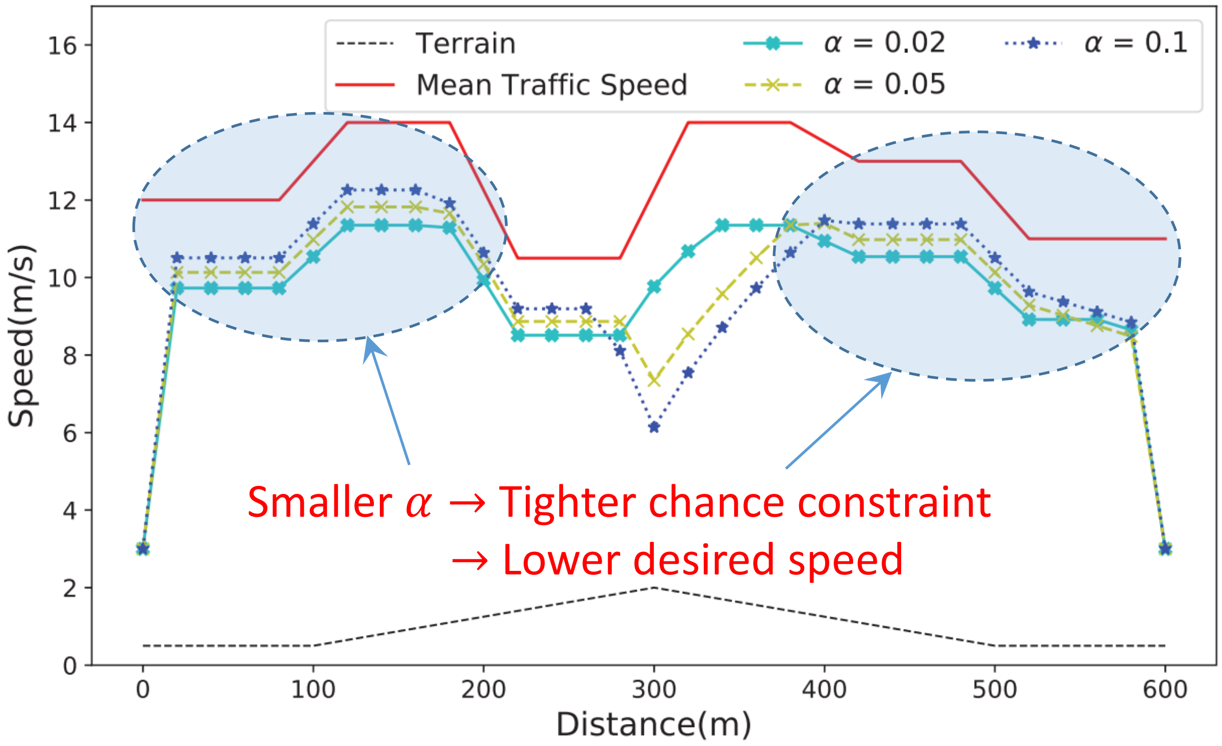
Relaxed chance constraint

Case Study: Stochastic Eco-driving Models

- Case setting
  - Distance $S = 600m$, discretized to 30 segments
  - Travel time budget $T = 65s$
  - Distribution of traffic speed: log-normal
  - Chance constraint $\alpha = 0.02, 0.05, 0.1$

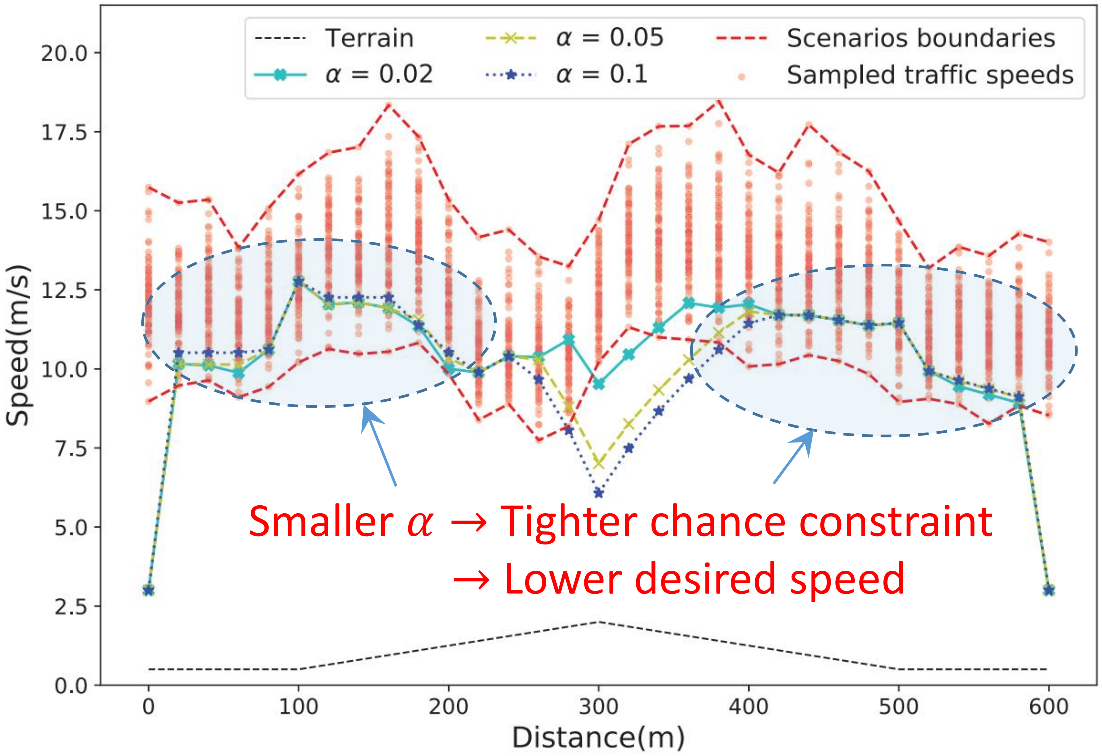
Case Study: Stochastic Eco-driving Model 1

- Chance constraint: $Prob(v_k \leq \bar{V}_k) \geq 1 - \alpha, \forall k$
- Computing time: 0.3s



Case Study: Stochastic Eco-driving Model 2

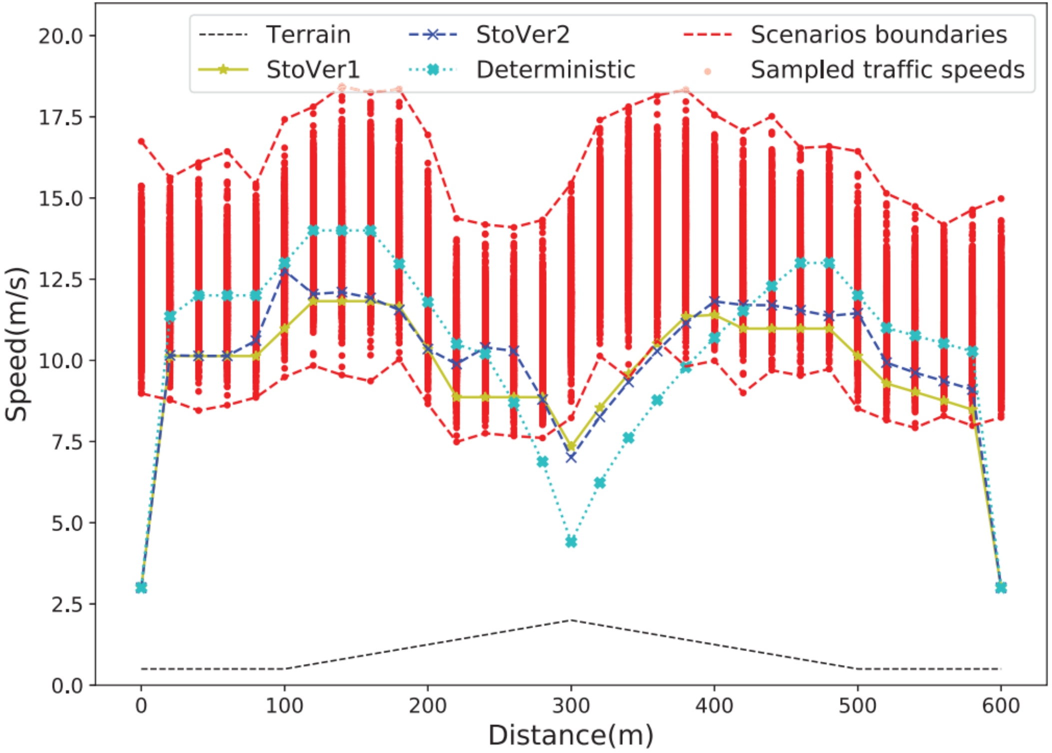
- Chance constraint: $Prob\left(\sum_{k=0}^{n-1} \frac{\Delta s}{v_k^{real}} \leq T\right) \geq 1 - \alpha$
- SAA (sample average approximation): 100 scenarios
- Computing time: $220s \sim 350s$



Case Study: Benefit of Stochastic Eco-driving Models

- Deterministic eco-driving is solved using mean traffic speed

Speed trajectories obtained by different eco-driving models, under $\alpha = 0.05$



Case Study: Benefit of Stochastic Eco-driving Models

- We generate 1000 new scenarios of actual traffic speed
- Under each of these 1000 scenarios, calculate the actual trip time and actual fuel consumption of the speed profiles generated by different models

❖ Average fuel consumption (g) over 1000 scenarios

α	Deter.	Stochastic Model 1	Stochastic Model 2
0.02	126.0	129.9	125.5
0.05	126.0	120.3	118.9
0.10	126.0	117.9	117.5

❖ Percentage of trips exceeding travel time budget over 1000 scenarios

α	Deter.	Stochastic Model 1	Stochastic Model 2
0.02	48.9%	35.0%	0
0.05	48.9%	55.6%	0
0.10	48.9%	78.2%	1.3%

Conclusions

- Solved the deterministic eco-driving problem much more efficiently by converting the non-convex program to a mixed-integer linear program
- Proposed two stochastic optimization formulations for the eco-driving problem under uncertain traffic speed
- Stochastic eco-driving models can mitigate the impact of uncertain traffic speed on eco-driving. It leads to lower fuel consumption and/or lower frequency of trip time violation, compared with the deterministic eco-driving model

Reference

Wu, F., Bektaş, T., Dong, M., Ye, H., Zhang, D., 2021. Optimal driving for vehicle fuel economy under traffic speed uncertainty. Transportation Research Part B 154, 175-206.